Frequency Domain Evaluation of Transient Finite Element Simulations of Induction Machines

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Abstract — A comprehensive procedure for the numerical evaluation of transient finite element simulations of induction machines in the frequency domain is presented. The proposed algorithm allows the determination of magnetic air-gap flux density and rotor current harmonics for arbitrary operational points under consideration of the rotor movement. The evaluation of the flux density harmonics can be performed on the stator as well as on the moving rotor surface. For skewed induction machines, an interpolation algorithm for the axial behavior of the flux density is included. The procedure will be demonstrated for a skewed induction machine operating at nominal point.

I. INTRODUCTION

Transient numerical simulation routines, mainly based on the finite element method (FEM), see e.g. [1], have found a large field of application in electrical machines. Most applications, such us the validation of machine design processes or the computation of audible noise etc., require a frequency domain evaluation of the transient numerical steady state results, in terms of rotating field waves of the magnetic air-gap fields or forces. For induction machines, the common straight-forward frequency domain evaluation applying a 2D fast Fourier transform (FFT) achieves rather poor accuracy in the frequency domain due to leakage effects [2]. Since, due to the high computational costs, it is generally not feasible to step through the entire very long steady state period resulting from the asynchronous rotor movement, these leakage effects cannot be avoided. Hence, a method for eliminating the leakage effects in the frequency domain and thus allowing the evaluation of the magnetic air-gap fields on the non-moving stator surface has been developed in an earlier work [3]. Unfortunately, due to a large number of close but distinct spectral lines, the method fails to determine the correct frequency spectrum on the moving rotor surface. Therefore, a modified algorithm is presented in this work, enhancing the accuracy of the former method regarding the estimated flux density components on the stator surface, and additionally allowing the determination of the corresponding harmonics on the air-gap surface of the rotor core stack. Beside this, a generalization of the present method will enable the precise estimation of rotor current harmonics in case of squirrel cage induction machines.

In the common case of skewed induction machines, as shown in [4], the axial behavior of the flux density or force waves has also to be regarded. Therefore, the method of [4] has been incorporated and further extended in this work, allowing a complete evaluation of the air-gap flux density and force harmonics in the frequency domain.

II. COMPUTATION OF AIR GAP FLUX DENSITY HARMONICS

The magnetic air-gap flux density in induction machines is commonly described in terms of a sum of rotating flux density waves. Considering a non-moving stator coordinate system (SCS) as well as a corresponding moving rotor coordinate system (RCS), a particular magnetic flux density wave $B^S(r,\omega^S)$ in the SCS and $B^R(r,\omega^S)$ in the RCS, with the spatial azimuthal order r and the angular frequency ω^S in the SCS or ω^R in the RCS, respectively, can be defined as

$$\begin{aligned} B^S(r,\omega^S) &= B_{r\omega}\cos(r\varphi-\omega^S t-\varphi_{r\omega}) \\ B^R(r,\omega^R) &= B_{r\omega}\cos(r\varphi-\omega^R t-\varphi_{r\omega}) \end{aligned}, \tag{1}$$

where φ is the azimuthal coordinate, t is time, $B_{r\omega}$ and $\varphi_{r\omega}$ correspond to amplitude and phase value of the flux density wave. Depending on the mechanical rotor speed corresponding to the slip value s of the operating point, the relation between the RCS and SCS can be expressed directly as

$$\omega^R = \omega^S - (1-s)\frac{r}{p}\omega_1, \tag{2}$$

where p is the number of pole pairs of the machine and $\omega_1=2\pi f_1$ denotes the line frequency. Although the occurring spatial orders r and frequencies ω^S depend strongly on machine design, all possible values can be predicted exactly by analytical approaches, see e.g. [5], due to the natural geometric periodicity in rotating electrical machines. Assuming a squirrel cage induction machine with a three phase integer slot winding on the stator and impressed stator currents with frequency f_1, all occurring flux density waves characterized by r and ω^S can be written as

$$\begin{aligned} r(g_1,g_2,k) &= p(1+6g_1)+g_2N_2+2kp \\ \omega^S(g_1,g_2,k) &= \omega_1\left[1+2k+g_2\frac{N_2}{p}(1-s)\right] \\ g_1,g_2 &= \ldots,-2,-1,0,1,2,\ldots \qquad k=1,2,3,(4,\ldots) \end{aligned}, \tag{3}$$

where N_2 is the rotor slot number and g_1, g_2 as well as k represent three independent indices related directly to the stator slotting and winding effects (g_1), the rotor slotting and winding effects (g_2) and the saturation effects (k). Whereas g_1 can take on larger values, too, the relevant ranges of g_2 and k are usually rather small.

Therewith, the rotating flux density waves characterized by $B_{r\omega}$ and $\varphi_{r\omega}$ for any given transient simulation results $B^S(\varphi,t)$ on the stator surface or $B^R(\varphi,t)$on the rotor surface, can be determined accurately in two steps. First, a Fourier transform from the (φ,t)- into (r,t)-domain, applying FFT, is performed. The real valued time domain signal of the flux density will lead to a symmetric spectrum in the (r,t)-domain with complex spectral lines $\underline{B}_r = \underline{B}_{-r}{}^*$ where '*' denotes complex conjugation.

In the second step, each occurring spectral line $\underline{B}_r$ with the ordinal number r is split up independently into all frequency values $\omega_{nr}{}^S$ or $\omega_{nr}{}^R$, respectively, estimated by (3) for the particular value of r

$$\begin{aligned} \underline{B}_r^S(r,t) &= \sum_n \underline{a}_{r\omega}\cos(\omega_{nr}^S t) + j\,\underline{b}_{r\omega}\sin(\omega_{nr}^S t) \\ \underline{B}_r^R(r,t) &= \sum_n \underline{a}_{r\omega}\cos(\omega_{nr}^R t) + j\,\underline{b}_{r\omega}\sin(\omega_{nr}^R t) \end{aligned}, \tag{4}$$

where j denotes the imaginary unit. The determination of the unknown complex coefficients $\underline{a}_{r\omega}$ and $\underline{b}_{r\omega}$ is best accomplished by a least square algorithm rather than a second FFT, in order to avoid leakage and aliasing effects. Note that for an estimated total number N_t of time steps calculated by the transient FEM simulation procedure, N_t spectral lines for each spatial order r can be obtained where these N_t frequency values $\omega_{nr}{}^S$ or $\omega_{nr}{}^R$, respectively, are allowed to be determined independently for each r using (3). An advantage of this procedure is that, since a particular spatial order r does not occur for any possible frequency $\omega^S(g_1,g_2,k)$, more than N_t frequency values in the (r,ω)-domain can be determined. In fact, this is also the reason why, contrary to the method of [3], this procedure is able to evaluate the rotating field waves on the moving rotor surface. Indeed, the corresponding frequencies $\omega_{nr}{}^R$ for a single spatial order r are clearly separated, whereas many frequencies of the overall set ω^R lay very close to each other.

Finally, the amplitude and phase values $B_{r\omega}$ and $\varphi_{r\omega}$, describing the rotating magnetic flux density waves can be determined by

$$\begin{aligned} B_{r\omega}e^{-j\varphi_{r\omega}} &= \underline{a}_{r\omega} + \underline{b}_{r\omega} \\ B_{(-r)\omega}e^{j\varphi_{(-r)\omega}} &= \underline{a}_{r\omega} - \underline{b}_{r\omega} \end{aligned}. \tag{5}$$

Beside the evaluation of the magnetic flux density, the magnetic air-gap forces as well as the rotor current harmonics in the squirrel cage can also be evaluated using the present method, if (3) is adapted accordingly.

In case of skewed induction machines, the axial behavior of the harmonics has also to be considered, since the direct interpolation along the axial sample points can lead to significant inaccuracies for some of the estimated harmonics. A corresponding approach, presented in [4] has been extended and incorporated into this work.

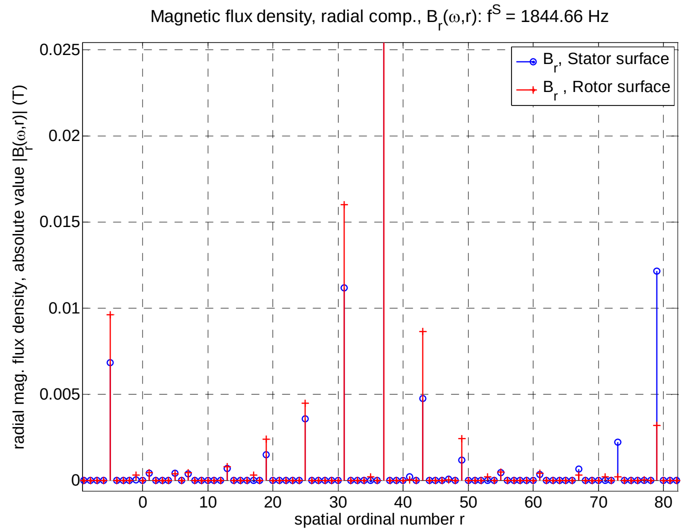

Fig. 1. Radial component of the magnetic flux density on the stator- and rotor air gap surface in *(r,ω)*-domain. The figure shows the spectral lines for f^S = 1844.66Hz.

III. PRELIMINARY RESULTS

The presented method has been applied to evaluate the numerical results of a skewed induction machine obtained by a transient FEM simulation using a multi-slice model (see e.g. [6]) of the machine. Figure 1 shows some preliminary results of the estimated magnetic flux density harmonics evaluated on the stator as well as on the moving rotor air-gap surface. The frequencies ω^R have been expressed in the SCS using (2) in order to allow a direct comparison of the corresponding field amplitudes on the stator and rotor surface.

IV. REFERENCES

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